IS 6200-1 (2003): Statistical Tests of Significance, Part 1: Normal t- and F-Tests [MSD 3: Statistical Methods for Quality and Reliability]

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भारतीय मानक

सार्थकता के लिए सांख्यिकीय परीक्षण

भाग 1 नार्मल, टी- और एफ-परीक्षण

(तीसरा पुनरीक्षण)

Indian Standard

STATISTICAL TESTS OF SIGNIFICANCE

PART 1 NORMAL, *t*- AND *F*- TESTS

(Third Revision)

ICS 03.120.30

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Statistical Method for Quality and Reliability Sectional Committee, MSD 3

FOREWORD

This Indian Standard (Part 1) (Third Revision) was adopted by the Bureau of Indian Standards, after the draft finalized by the Statistical Method for Quality and Reliability Sectional Committee had been approved by the Management and Systems Division Council.

This standard was originally published in 1971 and covered the industrial applications of three main tests of significance, namely, *t*-test, *F*-test and χ^2-test. It was then revised in 1977 into four parts to include tests for normality as also some non-parametric tests, which have wide application in industry.

The second revision of this part had been taken up in 1995 to rearrange the contents of the standard by putting together tests of the same hypothesis under different assumptions and requiring different use of test statistics. In addition, the revised standard includes: (a) the basic concepts of formation of null and alternative hypothesis; (b) Fisher-Behren's test for testing equality of means of two populations when the variances are not known and not assumed to be equal; and (c) examples on testing for proportions.

The third revision of the standard has been undertaken to:

a) modify the examples so as to make them practical and also that the reported test results to be of same decimal places as actually obtained in practice,
b) include the table for finding θ from tan θ to be used in Fisher-Behren's test, and
c) incorporate many editorial corrections.

The tests of significance described in this standard are useful in many problems of industrial experimentation. However, they should not be generally used for lot acceptance purposes for which IS 2500 (Part 1) : 2001 'Sampling inspection procedures : Part 1 Attribute sampling plans indexed by acceptable quality level (AQL) for lot-by-lot inspection' and IS 2500 (Part 2) : 1965 'Sampling inspection procedures : Part 2 Inspection by variables for percent defective' may be referred to.

In addition to this Part 1, IS 6200 has following three parts:

Part 2 χ^2-test
Part 3 Tests for normality
Part 4 Non-parametric tests

The composition of the Committee responsible for the formulation of this standard is given in Annex F.

Indian Standard

STATISTICAL TESTS OF SIGNIFICANCE

PART 1 NORMAL, *t*- AND *F*-TESTS

(Third Revision)

1 SCOPE

1.1 This standard (Part 1) lays down the following tests of significance:

- a) *One-sample test* — Testing of mean of a population against a specified value;
  - i) when the population variance is known, and
  - ii) when the population variance is not known.
- b) *Two-samples test* — Testing equality of means of two populations;
  - i) when the variances are known, and
  - ii) when the variances are not known.
- c) Testing for equality of variances of two populations; and
- d) Testing for proportions.

1.2 This standard does not include the *t*-tests for regression coefficient and correlation coefficient as they are covered in IS 7300 : 1995 'Methods of regression and correlation (*first revision*)'.

2 REFERENCES

The following standards contain provisions, which through reference in this text constitute provisions of this standard. At the time of publication, the editions indicated were valid. All standards are subject to revision and parties to agreements based on this standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below:

IS No.	*Title*
7920	Statistical vocabulary and symbols:
(Part 1) : 1994	Probability and general statistical terms (*second revision*)
(Part 2) : 1994	Statistical quality control (*second revision*)

3 TERMINOLOGY

For the purpose of this standard, the definitions given in IS 7920 (Part 1) and IS 7920 (Part 2) shall apply.

4 BASIC CONCEPTS

4.1 Statistical tests of significance are important tools in decision-making. They are extremely useful in finding out whether, in the case of one population, the mean value differs significantly from certain specified value or whether, in the case of two populations, the mean values differ significantly from each other. Thus, it may be desirable to find out whether a new germicide is more effective in treating a certain type of infection than a standard germicide, whether a new method of sealing light bulbs will increase their life or whether one method of preserving foods is better than another in so far as the retention of vitamins is concerned. In such cases, it would be necessary to examine whether the mean values obtained can be deemed as same or different. There may also be cases where it may be worthwhile to find out whether one inspector is more consistent than another or whether a new source of raw material has resulted in a change in the variability of the output or whether the temperature of the bath in which the cocoons are cooked affects the uniformity of the quality of silk. In these cases it will be necessary to determine whether the variances obtained are the same or not.

4.2 Formulation of Hypothesis

For taking a decision using statistical tests of significance, the first step is to form the hypotheses, namely, Null hypothesis (H_0) and alternative hypothesis (H_1).

4.2.1 *Null Hypothesis* (H_0)

The procedure commonly used is to first set up a null hypothesis regarding equivalence (no difference) of the assumed population mean and the specified value. The question on which the decision is called for, by applying the tests of significance, is translated in terms of null hypothesis in such a way that this null hypothesis would likely be rejected if there is enough evidence against it as seen from the data in the sample. For example, in the case of new germicide, a null hypothesis will be that it is not more effective than a standard germicide; or in the case of light bulbs the new method of sealing does not increase the life of the bulb.

4.2.2 *Alternative Hypothesis* (H_1)

Alternative hypothesis is opposite to the null hypothesis. It may be two-sided or one-sided.

4.2.2.1 *Two-sided hypothesis*

In some situations, it may be of interest to find out whether lot mean differs significantly from a specified value irrespective of the fact that this difference is positive or negative. For example, in the manufacture of certain cylindrical rods, one may have to examine whether average diameter differs significantly from the specified nominal value or one may wish to determine whether the night shift production differs from that of the day shift in respect of certain quality characteristics of the item. In such cases the test is said to be two-sided.

4.2.2.2 *One-sided hypothesis*

In some other situations, only the positive difference or the negative difference between the lot mean and specified value may be of interest. For example, when drinking water is tested for bacteria count, only high values of count may be the source of concern or when cases of concrete are tested for strength, low values may have to be detected. In such cases, the test is said to be one-sided.

4.3 Level of Significance

4.3.1 There are two kinds of errors involved in taking the decision based on the tests of significance, namely:

a) *Type I error* — Error in deciding that a significant difference exists when there is no real difference.

b) *Type II error* — Error in deciding that no difference exists when there is a real difference.

4.3.2 Type I error and Type II error is also called error of the first kind and error of the second kind respectively. This process of decision making is given below:

	H_0 True	H_1 True
Reject H_0	Type I error (α)	Correct decision
Accept H_0	Correct decision	Type II error (β)

4.3.3 Based on the distribution of test statistics used, it is possible to work out the probability of committing Type I error or Type II error. The probability of committing Type I error is called level of significance (α). It is not possible to minimize both these probabilities (risk) at the same time. Hence, one of the risks, usually of the first kind, is controlled by assigning it a chosen level of probability. Generally the value for level of significance is chosen as 0.05 or 0.01, that is, 5 percent or 1 percent. This implies confidence level of 95 percent or 99 percent respectively.

4.4 The decision making procedure involves the comparison of the calculated value of the statistic with the tabulated value. If the calculated value is greater than or equal to the tabulated value of the statistic, then H_0 is rejected, thereby accepting H_1; otherwise H_0 is not rejected. For practical purpose, H_0 not rejected is taken as, if it is accepted.

4.5 For each test of significance, certain underlying assumptions are made (*see* **5.1** and **8.2**). Hence, it is important that these tests are not used indiscriminately. If the assumptions are in doubt, it is advisable to obtain the guidance of a competent statistician to ascertain the feasibility of application of these tests.

5 ONE SAMPLE TEST — TESTING OF MEAN OF A POPULATION AGAINST A SPECIFIED VALUE

5.1 To judge whether the population mean (μ) differs significantly from a specified value, μ_0, a sample of size n is taken from the population and sample mean $\bar{x}$ is calculated.

In this case, null hypothesis is: $H_0 : \mu = \mu_0$

Depending upon the situation, any one of the following three alternative hypotheses may be selected:

a) $H_1 : \mu \neq \mu_0$ (two-sided),

b) $H_1 : \mu > \mu_0$ (one-sided), and

c) $H_1 : \mu < \mu_0$ (one-sided).

It is assumed that the observations follow normal distribution and are drawn at random. Depending upon the knowledge about the population variance, two cases may arise as given below.

5.2 Population Variance (σ^2) is Known

In this case the normal test is applied by computing the statistic

$$z = |\bar{x} - \mu_0| \sqrt{n}/\sigma \quad \text{for } H_1 : \mu \neq \mu_0$$
$$z = (\bar{x} - \mu_0) \sqrt{n}/\sigma \quad \text{for } H_1 : \mu > \mu_0$$
$$z = (\mu_0 - \bar{x}) \sqrt{n}/\sigma \quad \text{for } H_1 : \mu < \mu_0$$

The tabulated values of z are given in Annex A. These values will be used for taking the decision as per **4.4**.

5.2.1 *Example* 1

Ten samples from a consignment of resistors have the following values in kilo-ohms (k Ω):

96 108 94 98 102 91 92 104 105 101

The earlier measurements have shown that variation from consignment to consignment is stable and is represented by as standard deviation of 3kΩ. It is desired to find out whether the average value of the resistance for the consignment is:

a) equal to 100 k Ω, and

b) less than 100 k Ω.

Solution

From the 10 values of resistance, the average is obtained as $\bar{x} = 99.1$.

The null hypothesis is $H_0 : \mu = 100$.

a) In this case, the alternative hypothesis will be two-sided, given as, $H_1: \mu \neq 100$ The test statistic is computed as:

$$z = |x - \mu_0| \sqrt{n}/\sigma$$
$$= |99.1 - 100| \sqrt{10}/3 = 0.95$$

Since the calculated value is smaller than 1.96, the tabulated value of z at 5 percent level of significance for two-sided test (*see* Annex A), the null hypothesis is not rejected at 5 percent level of significance, that is, the resistance of the consignment is not different from 100 k Ω.

b) In this case, the alternative hypothesis will be one-sided, given as, $H_1: \mu < 100$. The test statistic is computed as:

$$z = (\mu_0 - \bar{x}) \sqrt{n}/\sigma = (100 - 99.1) \sqrt{10}/3 = 0.95$$

Since the calculated value is smaller than 1.645, the tabulated value of z at 5 percent level of significance for one-sided test (*see* Annex A), the null hypothesis is not rejected at 5 percent level of significance, that is, the resistance of the consignment is not less than 100 k Ω.

5.3 Population Variance is Unknown

5.3.1 In this case t-test is applied by computing the *statistic*,

$t = |\bar{x} - \mu_0| \sqrt{n}/s$ for $H_1 : \mu \neq \mu_0$
$t = (\bar{x} - \mu_0) \sqrt{n}/s$ for $H_1 : \mu > \mu_0$
$t = (\mu_0 - \bar{x}) \sqrt{n}/s$ for $H_1 : \mu < \mu_0$

where s^2 is the best unbiased estimate of σ^2 and is calculated as:

$$s^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / (n-1) = \left[\sum x_i^2 - n\bar{x}^2\right] / (n-1)$$

The t-test is used for small samples ($n \leq 30$). When the sample is large, t-test approximates to normal test (*see* **5.2**). The degrees of freedom for t is $(n - 1)$ which is one less than the number of observations. Tabulated values of t are given in Annex B. These values will be used for taking the decision as per **4.4**.

5.3.2 *Example* 2

The measurement of the diameters (in mm) of 15 pins manufactured by a machine gives the values as:

19.5	22.4	19.7	18.8	19.8	20.1	22.3	20.3
21.8	20.9	21.7	16.9	18.4	18.5	21.7	

It is required to find out whether the average diameter of the pins differs significantly from the specified diameter of 20.0 mm or not.

Solution

The mean ($\bar{x}$) and the standard deviation (s) of the sample measurements are found to be 20.19 and 1.62 mm respectively.

The null hypothesis is that the mean diameter of the pins manufactured by this machine is equal to 20.0 mm, that is, $H_0: \mu = 20.0$.

The alternative hypothesis is two-sided, given as, $H_1: \mu \neq 20.0$

The t-test is applied by calculating:

$$t = |\bar{x} - \mu_0| \sqrt{n}/s = |20.19 - 20.00| \sqrt{15}/1.62 = 0.454$$

Since the calculated value of t is less than 2.145, the tabulated value of t at 5 percent level of significance (two-sided) with 14 degrees of freedom (*see* Annex B), the validity of the null hypothesis is not rejected at 5 percent level of significance and the population mean diameter is taken to be in agreement with the specified diameter of 20.0 mm.

5.3.3 *Example* 3

A manufacturer produces a special alloy steel with an average tensile strength of 180.0 MPa. A change in the composition of the alloy is said to increase the tensile strength without affecting the variability. Ten tests done on the steel with the new composition gave the following values (in MPa):

178.2	180.5	181.8	182.4	182.8
183.1	183.4	183.8	183.9	185.6

It is required to find out whether the average tensile strength is significantly more than 180.0 MPa obtained for the earlier composition.

Solution

The mean ($\bar{x}$) and the standard deviation (s) of the sample test results are obtained as 182.55 and 2.04 respectively.

The null hypothesis (H_0) is that the mean tensile strength for the new composition is same as that for the earlier composition (equal to 180 MPa), against the alternative hypothesis that the new composition gives higher tensile strength than 180 MPa.

The t-test is applied by calculating:

$$t = (182.55 - 180.0) \sqrt{10}/2.04 = 3.95$$

Since the calculated value of t is more than the tabulated value of 1.833 at 5 percent level of significance with 9 degrees of freedom for one-sided test (*see* Annex B), the null hypothesis is rejected at

5 percent level of significance and it is concluded that the mean tensile strength of the steel with new composition is significantly higher than 180.0 MPa corresponding to the earlier composition.

6 TWO SAMPLES TEST — TESTING EQUALITY OF MEANS OF TWO POPULATIONS

6.1 To find out whether the means of two populations (μ_1 and μ_2) differ significantly from each other, two samples of sizes n_1 and n_2 are obtained from the two populations and their respective means $\bar{x}$ and $\bar{y}$ are calculated.

In this case, null hypothesis is H_0: $\mu_1 = \mu_2$

Depending upon the situation, any one of the following three alternative hypotheses may be selected:

a) $H_1 : \mu_1 \neq \mu_2$ (two-sided),
b) $H_1 : \mu_1 > \mu_2$ (one-sided), and
c) $H_1 : \mu_1 < \mu_2$ (one-sided).

Depending upon the knowledge about the variances of the two populations, two cases may arise as given below.

6.2 Population Variances are Known

In this case the normal test is applied by computing the statistic,

$$z = |\bar{x} - \bar{y}|/[(\sigma^2_1/n_1) + (\sigma_2^2/n_2)]^{1/2} \text{ for } H_1 : \mu_1 \neq \mu_2$$
$$z = (\bar{x} - \bar{y})/[(\sigma^2_1/n_1) + (\sigma_2^2/n_2)]^{1/2} \text{ for } H_1 : \mu_1 > \mu_2$$
$$z = (\bar{y} - \bar{x})/[(\sigma^2_1/n_1) + (\sigma_2^2/n_2)]^{1/2} \text{ for } H_1 : \mu_1 < \mu_2$$

Where σ_1 and σ_2 denote the known standard deviations for the two populations. $\bar{x}$ and $\bar{y}$ are the means of the samples of sizes n_1 and n_2 drawn from these populations respectively. The table values of z are given in Annex A. These values will be used for taking the decisions as per **4.4**.

6.2.1 *Example* 4

Ten samples of a particular type of yarn were tested from the consignment of manufacturer A whereas twelve sample results were available for manufacturer B. The breaking load (in Newton) of the samples tested is given in Table 1.

It is known that the standard deviations for breaking load have been satisfactorily established from the earlier measurements as 0.36 for both the manufacturers. It is desired to compare whether the mean breaking load of the yarn for:

a) two manufacturers is significantly different from each other.
b) manufacturer A is less than that for manufacturer B.

Table 1 Breaking Load Values
(*Clause* 6.2.1)

Manufacturer A (1)	Manufacturer B (2)
2.3	2.3
2.6	2.3
1.9	2.4
2.4	3.2
2.0	3.2
2.1	2.8
1.9	2.2
2.0	2.4
1.6	2.2
2.9	2.5
	2.1
	2.7

Solution

From the data, the average breaking load for the two manufacturers is obtained as:

Manufacturer A: $\bar{x} = 2.17$
Manufacturer B: $\bar{y} = 2.52$

The null hypothesis (H_0) is that the mean breaking load of the yarn for the two manufacturers is the same.

a) In this case, the alternative hypothesis is that the mean breaking load of the yarn for the two manufacturers is different. The test-statistic is computed as,

$$z = |\bar{x} - \bar{y}|/[(\sigma_1^2/n_1) + (\sigma_2^2/n_2)]^{1/2}$$
$$= |2.17 - 2.52|/[(0.36)^2/10) + (0.36)^2/12)]^{1/2}$$
$$= 2.59$$

Since this calculated value is greater than the tabulated value of 1.960 at 5 percent level of significance (for two-sided test) from Annex A, the null hypothesis, that the mean breaking load of the yarn for the two manufacturers is same, is rejected at 5 percent level of significance.

b) In this case, the alternative hypothesis is that the mean breaking load of the yarn for manufacturer A is lower than that of manufacturer B. The test statistic is computed as:

$$z = (\bar{y} - \bar{x})/[(\sigma^2_1/n_1) + (\sigma^2_2/n_2)]^{1/2} = 2.59$$

Since this is greater than 1.645 and 2.326, the tabulated values at 5 percent and 1 percent levels of significance respectively, for one-sided test (*see* Annex A), the null hypothesis that the mean breaking load of the yarn for the two manufacturers is same, is rejected at 5 percent as well as at 1 percent level of significance.

6.3 Population Variances are not Known

6.3.1 *Independent Populations with Variances Assumed to be Equal*

In this case, t-test is applied by computing the following statistic:

$$t = |\bar{x} - \bar{y}|/s'[(n_1 + n_2)/n_1 n_2]^{1/2} \quad \text{for } H_1 : \mu_1 \neq \mu_2$$
$$t = (\bar{x} - \bar{y})/s'[(n_1 + n_2)/n_1 n_2]^{1/2} \quad \text{for } H_1 : \mu_1 > \mu_2$$
$$t = (\bar{x} - \bar{y})/s'[(n_1 + n_2)/n_1 n_2]^{1/2} \quad \text{for } H_1 : \mu_1 < \mu_2$$

where, s' refers to the standard deviation of the pooled samples and is given by:

$$\begin{aligned} s' &= [\Sigma(x_i - \bar{x})^2 + \Sigma(y_i - \bar{y})^2]^{1/2}/(n_1 + n_2 - 2)^{1/2} \\ &= [\Sigma x_i^2 + \Sigma y_i^2 - n_1 \bar{x}^2 - n_2 \bar{y}^2]/(n_1 + n_2 - 2)^{1/2} \end{aligned}$$

This t-test should satisfy all the conditions mentioned in the case of one sample t-test (*see* **5.3**). Over and above, for two samples t-test, additional condition that the population variances are same should also be satisfied. The pooled variance s'^2 has, in fact, been calculated on the basis of the above assumption. Thus, it is necessary first to test the equality of variances (*see* **8**) before applying the t-test.

The t in this case has $(n_1 + n_2 - 2)$ degrees of freedom. The rules for rejection of the null hypothesis regarding the equality of two means are similar to those given in **5.3** for one sample. For large sample sizes, t-test approximates to Normal test.

6.3.1.1 *Example* 5

To verify the contention that for the characteristic of water soluble ash (expressed as percentage of total ash), the black tea (from Kangra valley) gives a higher value than the green tea (from Dehra Dun valley), data were collected pertaining to both these teas. The water soluble ash of 15 test results pertaining to black tea and 12 test results pertaining to green tea are given in Table 2.

Table 2 Water Soluble Ash for Two Varieties of Tea

Black Tea (1)	Green Tea (2)
40.4	40.0
41.9	43.4
50.0	48.0
55.9	60.9
33.9	44.8
46.0	42.1
44.8	44.7
50.2	47.2
48.0	40.5
36.3	40.1
49.0	38.0
51.8	46.2
47.9	–
51.9	–
45.8	–

For the black tea the mean $\bar{x}$ = 46.3 and the sample size n_1 = 15, whereas for the green tea the mean $\bar{y}$ = 44.7 and the sample size n_2 = 12. The pooled standard deviation s' for the two samples was calculated as = 6.0.

To test the null hypothesis that the mean value of water soluble ash for the two teas is the same against the alternative hypothesis that the water soluble ash for black tea is more than that for the green tea, t statistic is calculated as:

$$\begin{aligned} t &= (\bar{x} - \bar{y})/s'[(n_1 + n_2)/n_1 n_2]^{1/2} \\ &= (46.3 - 44.7)/6.0[(15 + 12)/(15 \times 12)] = 0.69 \end{aligned}$$

Since the calculated value of t is smaller than the tabulated value of 1.708 at 5 percent level of significance for 25 degrees of freedom (one-sided test) from Annex B, the null hypothesis is not rejected. Hence, the water soluble ash for the two tests is the same.

6.3.2 *Independent Populations with Variances not Assumed to be Equal*

In this case, Fisher-Behren's test is applied by computing the following statistic:

$$d = |\bar{x} - \bar{y}|/\{[s_1^2/(n_1 - 1)] + [s_2^2/(n_2 - 1)]\}^{1/2}$$
$$\text{Let } \tan^2\theta = s_2^2(n_1 - 1)/s_1^2(n_2 - 1)$$

where, s_1 and s_2 denote the standard deviations of the two samples of sizes n_1 and n_2 respectively.

The value of tan θ is obtained from the above relation. The value of θ from tan θ is then obtained from Annex C. The tabulated values of d for given values of θ, n_1 and n_2 are given in Annex D. These values will be used for taking the decision as per **4.4**.

6.3.3 *Related Populations* (*Paired t-Test*)

Sometimes two samples are so given that observations of one sample correspond to the observations of the other. Thus the observations may occur in pairs, each pair arising under the same experimental conditions with the conditions varying from pair to pair. In such a case the differences between the observations in each pair are taken and t-test is applied on these differences according to the procedure given in **5.3**. The null hypothesis is that the mean of the differences is zero against the alternative hypothesis that the mean of the difference is not equal to zero.

6.3.3.1 *Example* 6

The test results for compressive strength of 16 cements using two grades of standard sand are obtained as in col. 2 and 3 of Table 3. It is required to examine whether or not there is a significant effect of the grade of sand on the average compressive strength of cements.

Solution

$\Sigma d = -1.2$ and $\Sigma d^2 = 0.48$

$\bar{d} = \Sigma d/n = -0.08$, $s^2 = [\Sigma d^2 - (\Sigma d)^2/n]/(n-1)$
$= 0.026$;
$s = 0.16$

Table 3 Compressive Strength of Cement Using Two Grades of Sand

(*Clause* 6.3.3.1)

Cement No.	Compressive Strength for Sand		Differences, d = A – B
	A	B	(2) - (3)
(1)	(2)	(3)	(4)
1	24.1	24.0	0.1
2	23.2	23.4	–0.2
3	26.0	26.0	0.0
4	23.7	23.9	–0.2
5	26.5	26.5	0.0
6	27.0	27.0	0.0
7	27.1	26.9	0.2
8	23.5	23.9	–0.4
9	26.1	26.0	0.1
10	25.3	25.5	–0.2
11	26.3	26.5	–0.2
12	23.3	23.3	0.0
13	22.7	23.0	–0.3
14	25.8	25.8	0.0
15	27.5	27.5	0.0
16	22.2	22.3	–0.1

To examine if the average difference is significantly different from zero, the null hypothesis is H_0: $\bar{d} = 0$ against the alternative hypothesis, H_1: $d \neq 0$.

$$t = |\bar{d} - 0|\sqrt{n}/s = 0.08 \times \sqrt{16}/0.16 = 2.0$$

This value is less than 2.131, the tabulated value of t at 5 percent level of significance (two-sided) with 15 degrees of freedom (*see* Annex B). Hence the null hypothesis that the average compressive strength of the cement does not differ with the two grades of the standard sand is not rejected.

6.3.3.2 If the number of observations in the sample, n, becomes large, the t-test becomes equivalent to the Normal test. When the sample size is greater than 30, Normal test may be used instead of t-test for all practical purposes.

7 TESTING FOR PROPORTION

7.1 The normal test described in **5.2** and **6.2** may also be extended to the testing for proportions. If the binomial proportion p (obtained as the ratio of the number of successes in n repetitive trials) is to be tested against a specified value p_0 then the null hypothesis is H_0: $\pi = p_0$ against the three possible alternative hypotheses, namely,

H_1: $\pi \neq p_0$ (two-sided)
H_1: $\pi < p_0$ (one-sided)
H_1: $\pi > p_0$ (one-sided)

where π is the population proportion of success. Further, n should be sufficiently large, say more than 30. Then the z-statistic to be calculated is obtained as:

$$z = |\bar{p} - p_0| [p_0(1-p_0)/n]^{1/2} \quad \text{for } H_1 : \pi \neq p_0$$
$$z = (p_0 - \bar{p}) \mid [p_0(1-p_0)/n]^{1/2} \quad \text{for } H_1 : \pi < p_0$$
$$z = (p_0 - \bar{p}) \mid [p_0(1-p_0)/n]^{1/2} \quad \text{for } H_1 : \pi > p_0$$

The tabulated values of z are given in Annex A. These values will be used in taking the decision as per **4.4**.

7.1.1 *Example* 7

A purchaser picks up at random a sample of size 15 from a lot and finds one non-conforming item. It is required to test the hypothesis that the lot contains 5 percent non-conforming items.

Here, $\bar{p} = 1/15 = 0.067$; $n = 15$; $p_0 = 5/100 = 0.05$

To examine if the proportion non-conforming in the lot, p_0 is 0.05, the null hypothesis $H_0 : \pi = p_0$ is tested against the alternative hypothesis H_1: $\pi \neq p_0$

The z statistic is calculated as:

$$= |0.067 - 0.05|/[0.05\ (1 - 0.05)/15]^{1/2}$$
$$= 0.017/0.056 = 0.30$$

This value is less than 1.96, the tabulated value of z at 5 percent level of significance (two-sided) (*see* Annex A). Hence, the null hypothesis that the proportion non-conforming in the lot is 0.05 is not rejected.

7.2 In the case of testing the equality of two proportions corresponding to two populations, if p_1 and p_2 are the two proportions obtained on the basis of samples of size n_1 and n_2 drawn from the two populations respectively, then calculate:

$$p' = (n_1p_1 + n_2p_2)/(n_1 + n_2)$$

In this case, the null hypothesis is H_0: $\pi_1 = \pi_2$ against the alternative hypothesis $H_1 : \pi_1 \neq \pi_2$ or $H_1 : \pi_1 < \pi_2$ or $H_1 : \pi_1 > \pi_2$.

The z-statistic to be used is obtained as:

$$z = |\bar{p}_1 - \bar{p}_2|/[p'(1-p')(n_1 + n_2)/n_1n_2]^{1/2} \quad \text{for } H_1 : \pi_1 \neq \text{p}_2$$
$$z = (\bar{p}_1 - \bar{p}_2)/[p'(1-p')(n_1 + n_2)/n_1n_2]^{1/2} \quad \text{for } H_1 : \pi_1 > \pi_2$$
$$z = (\bar{p}_2 - \bar{p}_1)/[p'(1-p')(n_1 + n_2)/n_1n_2]^{1/2} \quad \text{for } H_1 : \pi_1 < \pi_2$$

The rules for rejecting or not rejecting the null hypothesis are similar to those in **7.1**.

7.2.1 *Example* 8

A machine puts out 20 imperfect articles in a sample of 500. After machine is overhauled, it produced 3 imperfect articles in a batch of 100. Has the machine improved?

Here, $n_1 = 500$, $\bar{p}_1 = 20/500 = 0.04$,

$n_2 = 100$, $\bar{p}_2 = 3/100 = 0.03$

The null hypothesis is H_0: $\pi_1 = \pi_2$ and the alternative hypothesis is $H_1 : \pi_1 > \pi_2$

$p' = (500 \times 0.04 + 100 \times 0.03)/(500 + 100) = 0.038$

The test statistic is computed as:

$$\begin{aligned} z &= (\bar{p}_1 - \bar{p}_2)/[p'(1-p')\,(n_1+n_2)/n_1n_2]^{1/2} \\ &= (0.04 - 0.03)/[0.038\ (1 - 0.038) \\ &\quad (500 + 100)/500 \times 100]^{1/2} = 0.01/0.021 \\ &= 0.48 \end{aligned}$$

This value is less than 1.645, the tabulated value of z at 5 percent level of significance (one-sided) (*see* Annex A). Hence the null hypothesis is not rejected, that is the machine has not improved.

8 TESTING EQUALITY OF VARIANCES

8.1 To test the equality of the variances of the two populations, say σ_1^2 and σ_2^2, null hypothesis is, $H_0 : \sigma_1^2 = \sigma_2^2$.

The alternative hypothesis may be one-sided or two-sided as:

$H_1 : \sigma_1^2 \neq \sigma_2^2$ (two-sided), and
$H_1 : \sigma_1^2 > \sigma_2^2$ (one-sided).

Thus, if the variances of the samples of sizes n_1 and n_2 from the two populations be s_1^2 and s_2^2, respectively, the following *statistic* is computed:

$$\begin{aligned} F &= s_1^2/s_2^2 \quad \text{when } H_1 : \sigma_1^2 \neq \sigma_2^2 \\ F &= s_1^2/s_2^2 \quad \text{when } H_1 : \sigma_1^2 > \sigma_2^2 \\ F &= s_1^2/s_2^2 \quad \text{when } H_1 : \sigma_1^2 < \sigma_2^2 \end{aligned}$$

where

$$\begin{aligned} s_1^2 &= \Sigma(x - \bar{x})^2/(n_1 - 1) = (\Sigma x^2 - n_1\bar{x}^2)/(n_1 - 1) \\ s_2^2 &= \Sigma(y - \bar{y})^2/(n_2 - 1) = (\Sigma y^2 - n_2\bar{y}^2)/(n_2 - 1) \end{aligned}$$

The larger of two variances is taken in the numerator. The value of F is associated with $(n_1 - 1)$, $(n_2 - 1)$ degrees of freedom when $s_1^2 > s_2^2$. In case s_2^2 is greater than s_1^2, the degrees of freedom would be $(n_2 - 1)$, $(n_1 - 1)$.

The tabulated values of F are given in Annex E. These values will be used for taking the decision as per **4.4**.

8.1.1 *Example* 9

There are two alternative methods X and Y available for making a chemical determination in parts per million (ppm). The results of 20 determinations by Method X on a given sample and 15 determinations by Method Y on the same sample are given in Table 4. It is required to find whether any one method gives more consistent results:

Solution

The variances of the two sets of results as indicated in **8.1** are obtained as

s_1^2 = 0.062 for Method X, and
s_2^2 = 0.019 for Method Y.

Table 4 Chemical Determination by Two Methods

(*Clause* 8.1.1)

Method *X*	**Method *Y***
(1)	(2)
1.00	0.75
0.95	0.70
1.45	0.70
0.70	0.95
1.05	0.75
0.95	0.90
1.15	1.00
0.95	1.05
1.35	1.10
1.10	0.95
1.50	0.85
0.80	0.80
0.95	0.70
1.10	1.05
0.85	0.85
1.30	
0.75	
0.90	
1.55	
1.25	

In this case, the null hypothesis is that the variations in results of the chemical determination by the two methods are the same against the alternative hypothesis that one method gives more consistent results than the other.

Applying the F-test,

$$F = s_1^2/s_2^2 = 0.062/0.019 = 3.235$$

Since the calculated value of F is greater than 2.40 which is the tabulated value at 5 percent level of significance with 19 and 14 degrees of freedom in the Table in Annex E (for one-sided test), the null hypothesis is rejected. Further, since s_2^2 is smaller than s_1^2, it is concluded that the Method Y gives more consistent results as compared to Method X.

8.2 Like t-test, the F-test is also based on the assumption of the normality and independence of the observations (*see also* **4.4**).

ANNEX A

(*Clauses* 5.2, 5.2.1, 6.2, 6.2.1, 7.1.1 *and* 7.2.1)

CRITICAL VALUES OF STANDARD NORMAL DISTRIBUTION (z)

Significance Level (α)	*One- Sided Test*	*Two-Sided Test*
(1)	(2)	(3)
0.05	1.645	1.960
0.01	2.326	2.576

ANNEX B

(*Clauses* 5.3.1, 5.3.2, 5.3.3, 6.3.1.1 *and* 6.3.3.1)

CRITICAL VALUES OF t-DISTRIBUTION

Degrees of Freedom	*One-Sided Test Significance Levels*		*Two-Sided Test Significance Levels*	
	5 percent	1 percent	5 percent	1 percent
(1)	(2)	(3)	(4)	(5)
1	6.314	31.821	12.706	63.657
2	2.920	6.965	4.303	9.925
3	2.353	4.541	3.182	5.841
4	2.132	3.747	2.776	4.604
5	2.015	3.365	2.571	4.032
6	1.943	3.143	2.447	3.707
7	1.895	2.998	2.365	3.499
8	1.860	2.896	2.306	3.355
9	1.833	2.821	2.262	3.250
10	1.812	2.764	2.228	3.169
11	1.796	2.718	2.201	3.106
12	1.782	2.681	2.179	3.055
13	1.771	2.650	2.160	3.012
14	1.761	2.624	2.145	2.977
15	1.753	2.602	2.131	2.947
16	1.746	2.583	2.120	2.921
17	1.740	2.567	2.110	2.898
18	1.734	2.552	2.101	2.878
19	1.729	2.539	2.093	2.861
20	1.725	2.528	2.086	2.845
21	1.721	2.518	2.080	2.831
22	1.717	2.508	2.074	2.819
23	1.714	2.500	2.069	2.807
24	1.711	2.492	2.064	2.797
25	1.708	2.485	2.060	2.787
26	1.706	2.479	2.056	2.779
27	1.703	2.473	2.052	2.771
28	1.701	2.467	2.048	2.763
29	1.699	2.462	2.045	2.756
30	1.697	2.457	2.042	2.750
31	1.645	2.326	1.960	2.756

ANNEX C

(*Clause* 6.3.2)

VALUES OF NATURAL TANGENTS

Degrees	*0′*	*6′*	*12′*	*18′*	*24′*	*30′*	*36′*	*42′*	*48′*	*54′*	*Mean Differences*				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
0	.00000	00175	00349	00524	00698	00873	01047	01222	01396	01571	29	58	87	116	146
1	.01746	01920	02095	02269	02444	02619	02793	02968	03143	03317	29	58	87	116	146
2	.03492	03667	03842	04016	04191	04366	04541	04716	04891	05066	29	58	87	117	146
3	.05241	05416	05591	05766	05941	06116	06291	064767	06642	06817	29	58	88	117	146
4	.06993	07168	07344	07519	07695	07870	08046	08221	08397	08573	29	59	88	117	146
5	.08749	08925	09101	09277	09453	09629	09805	09981	10158	10334	29	59	88	117	147
6	.10510	10687	10863	11040	11217	11394	11570	11747	11924	12101	29	59	88	118	147
7	.12278	12456	12633	12810	12988	13165	13343	13521	13698	13786	30	59	89	118	148
8	.14054	14232	14410	14588	14767	14945	15124	15302	15481	15660	30	59	89	119	149
9	.15838	16017	16196	16376	16555	16734	16914	17093	17273	17453	30	60	90	120	150
10	.17633	17813	17993	18173	18353	18534	18714	18895	19076	19257	30	60	90	120	150
11	.19438	19619	19801	19982	20164	20345	20527	20709	20891	21073	30	60	91	121	152
12	.21256	21438	21621	21804	21986	22169	22353	22536	22719	22903	30	61	92	122	153
13	.23087	23271	23455	23639	23823	24008	24193	24377	24562	24747	31	61	93	124	155
14	.24933	25118	25304	25490	25676	25862	26048	26235	26421	26608	31	62	93	124	155
15	.26795	26982	27169	*27357*	27545	27732	27920	28109	28297	28486	31	63	94	125	157
16	.28675	28864	29053	29242	29432	29621	29811	30001	30192	30382	32	63	95	127	158
17	.30573	30764	30955	31147	31338	31530	31722	31914	32106	32299	32	64	96	128	160
18	.32492	32685	32878	33072	33266	33460	33654	33848	34043	34238	32	65	97	129	162
19	.34433	34628	34824	35019	35216	35412	35608	35805	36002	36199	33	66	98	131	164
20	.36397	36595	36793	36991	37190	37388	37588	37787	37986	38186	33	66	99	133	166
21	.38386	38587	38787	38988	39190	39391	39593	39795	39997	40200	34	67	101	134	168
22	.40403	40606	40809	41013	41217	41421	41626	41831	42036	42242	34	68	102	136	170
23	.42447	42654	42860	43067	43274	43481	43689	43897	44105	44314	34	69	104	138	173
24	.44523	44732	44942	45152	45362	45573	45784	45995	46206	46418	35	70	105	141	176
25	.46631	46843	47056	47270	47483	47698	47912	48127	48342	48557	36	71	107	143	179
26	.48773	48989	49206	49423	49640	49858	50076	50295	50514	50733	36	73	109	145	182
27	.50953	51173	51393	51614	51835	52057	52279	52501	52724	52947	37	74	111	148	185
28	.53171	52395	53620	53844	54070	54296	54522	54748	54975	55203	38	75	113	151	188
29	.55431	55659	55888	56117	56347	56577	56808	57039	57271	57503	38	77	115	154	192
30	.57735	57968	58201	58435	58670	58905	59140	59376	59612	59849	39	78	118	157	196
31	.60086	60324	60562	60801	61040	61280	61520	61761	62003	62245	40	79	120	160	200
32	.62487	62730	62973	63217	63462	63707	63953	64199	64446	64693	41	82	123	164	205
33	.64941	65189	65438	65688	65938	66189	66440	66692	66944	67197	42	84	126	167	209
34	.67451	67705	67960	68215	68471	68728	68985	69243	69502	69761	43	86	129	171	214
35	.70021	70281	70542	70804	71066	71329	71593	71857	72122	72388	44	88	132	176	219
36	.72654	72921	73189	73457	73726	73996	74267	74538	74810	75082	45	90	135	180	225
37	.75355	75629	75904	76180	76456	76733	77010	77289	77568	77848	46	92	139	185	231
38	.78129	78410	78692	78975	79259	79544	79829	80115	80402	80690	47	95	142	190	237
39	.80978	81268	81558	81849	82141	82434	82727	83022	83317	83613	49	98	147	195	244
40	.83910	84208	84507	84806	85107	85408	85710	86014	86318	86623	50	100	151	201	252

ANNEX C (*Continued*)

Deg-rees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	*Mean Differences* 1	 2	 3	 4	 5
41	.86929	87236	87543	87852	88162	88473	88784	89097	89410	89725	52	103	155	207	259
42	.90040	90357	90674	90993	91313	91633	91955	92277	92601	92926	53	107	160	214	268
43	.93252	93578	93906	94235	94565	94896	95229	95562	95897	96232	55	111	165	221	276
44	.96569	96907	97246	97586	97927	98270	98613	98958	99304	99652	57	114	171	229	286
45	1.00000	00350	00701	01053	01406	01761	02117	02474	02832	03192	58	118	177	237	296
46	1.03553	03915	04279	04644	05010	05378	05747	06117	06489	06862	61	123	184	245	307
47	1.07237	07613	07990	08369	08749	09131	09514	09899	10285	10672	63	127	191	255	319
48	1.11061	11452	11844	12238	12633	13029	13428	13828	14229	14632	66	132	199	265	331
49	1.15037	15443	15851	16261	16672	17085	17500	17916	18334	18754	69	138	207	276	344
50	1.19175	19599	20024	20451	20879	21310	21742	22176	22612	23050	72	143	216	288	359
51	1.23490	23931	24375	24820	25268	25717	26169	26622	27077	27535	75	150	225	300	375
52	1.27994	28456	28919	29385	29853	30323	30795	31269	31745	32224	78	157	235	314	392
53	1.32704	33187	33673	34160	34650	35142	35637	36134	36633	37134	82	164	247	329	411
54	1.37638	38145	38653	39165	39679	40195	40714	41235	41759	42286	86	172	259	345	431
55	1.42815	43347	43881	44418	44958	45501	46046	46595	47146	47700	91	181	272	362	453
56	1.48256	48816	49378	49944	50512	51034	51658	52235	52816	53400	95	191	286	382	477
57	1.53987	54576	55170	55767	56366	56969	57575	58184	58797	59414	100	201	302	403	504
58	1.60033	60657	61283	61914	62548	63185	63826	64471	65120	65772	106	213	319	426	533
59	1.66428	67088	67752	68419	69091	69766	70446	71129	71817	72509	113	226	339	452	564
60	1.73205	73905	74610	75319	76032	76749	77471	78198	78929	79665	120	240	360	481	600
61	1.80405	81150	81900	82654	83413	84177	84946	85720	86500	87283	128	255	383	511	639
62	1.88073	88867	89667	90472	91282	92098	92920	93746	94579	95417	136	273	409	546	683
63	1.96261	97111	97967	98828	99695	2.00569	2.01449	2.02335	2.03227	2.04125	146	292	438	584	731
64	2.05030	05942	06860	07785	08716	09654	10600	11552	12511	13477	157	314	471	629	786
65	2.14451	15432	16420	17416	18419	19430	20449	21475	22510	23553	169	338	508	677	846
66	2.24604	25663	26730	27806	28891	29984	31086	32197	33317	34447	183	366	549	732	915
67	2.35585	36733	37891	39058	40235	41421	42618	43825	45043	46270	199	397	596	795	994
68	2.47509	48758	50018	51289	52571	53865	55170	56487	57815	59156	Mean differences cause to be sufficiently accurate				
69	2.60509	61874	63252	64642	66046	67462	68892	70335	71792	73263					
70	2.74748	76247	77761	79289	80833	82391	83965	85556	87161	88783					
71	2.90421	92076	93748	95437	97144	98868	3.00611	3.02372	3.04152	3.05950					
72	3.07768	09606	11464	13341	15240	17159	19100	21063	23048	25055					
73	3.27085	29139	31216	33317	35443	37594	39771	41973	44202	46458					
74	3.48741	51053	53393	55761	58160	60588	63048	65538	68061	70616					
75	3.73205	75828	78485	81177	83906	86671	89474	92316	95196	98117					
76	4.01078	04081	07127	10216	13350	16530	19756	23030	26352	29724					
77	4.33148	36623	40152	43735	47374	51071	54826	58641	62518	66458					
78	4.70463	74534	78673	82882	87162	91516	95945	5.00451	5.05037	5.09704					
79	5.14455	19293	24218	29235	34345	39552	44857	50264	55777	61397					
80	5.67128	72974	78938	85024	91236	97576	6.04051	6.10664	6.17419	6.24321					

ANNEX C (*Concluded*)

Deg-rees	*0′*	*6′*	*12′*	*18′*	*24′*	*30′*	*36′*	*42′*	*48′*	*54′*	*Mean Differences*				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
81	6.31375	38587	45961	53503	61220	69116	77199	85475	93952	7.02637					
82	7.11537	20661	30018	39616	49465	59575	69957	80622	91582	8.02848					
83	8.14435	26356	38625	51259	64275	77689	91520	9.05789	9.20516	9.35724					
84	9.51436	9.6768	9.8448	10.019	10.199	10.385	10.579	10.780	10.988	11.205					
85	11.430	11.664	11.909	12.163	12.429	12.706	12.996	13.300	13.617	13.951					
86	14.301	14.669	15.056	15.464	15.895	16.350	16.832	17.343	17.886	18.464					
87	19.081	19.740	20.446	21.205	22.022	22.904	23.859	24.898	26.031	27.271					
88	28.636	30.145	31.821	33.694	35.801	38.188	40.917	44.066	47.740	52.081					
89	57.290	63.657	71.615	81.847	95.489	114.60	143.24	190.98	286.48	572.96					
90	∞														

ANNEX D

(*Clause* 6.3.2)

SIGNIFICANCE OF DIFFERENCE BETWEEN TWO MEANS

(P.V. Sukhatme)

	n_1	*0°*	*15°*	*30°*	*45°*	*60°*	*75°*	*90°*
5 percent points	6	2.447	2.440	2.435	2.435	2.435	2.440	2.447
	8	2.447	2.430	2.398	2.364	2.331	2.310	2.306
$n_2 = 6$	12	2.447	2.423	2.367	2.301	2.239	2.193	2.179
	24	2.447	2.418	2.342	2.247	2.156	2.088	2.064
	∞	2.447	2.413	2.322	2.201	2.082	1.993	1.960
	6	2.306	2.310	2.331	2.364	2.398	2.430	2.447
	8	2.306	2.300	2.294	2.292	2.294	2.300	2.306
$n_2 = 8$	12	2.306	2.292	2.262	2.229	2.201	2.183	2.179
	24	2.306	2.286	2.236	2.175	2.118	2.077	2.064
	∞	2.306	2.281	2.215	2.128	2.044	1.982	1.960
	6	2.179	2.193	2.239	2.301	2.367	2.423	2.447
	8	2.179	2.183	2.201	2.229	2.262	2.292	2.306
$n_2 = 12$	12	2.179	2.175	2.169	2.167	2.169	2.175	2.179
	24	2.179	2.168	2.142	2.112	2.085	2.069	2.064
	∞	2.179	2.163	2.120	2.064	2.011	1.973	1.960
	6	2.064	2.088	2.156	2.247	2.342	2.418	2.447
	8	2.064	2.077	2.118	2.175	2.236	2.286	2.306
$n_2 = 24$	12	2.064	2.069	2.085	2.112	2.142	2.168	2.179
	24	2.064	2.062	2.058	2.056	2.058	2.062	2.064
	∞	2.064	2.056	2.035	2.009	1.983	1.966	1.960
	6	1.960	1.993	2.082	2.201	2.322	2.413	2.447
	8	1.960	1.982	2.044	2.128	2.215	2.281	2.306
$n_2 = \infty$	12	1.960	1.973	2.011	2.064	2.120	2.163	2.179
	24	1.960	1.966	1.983	2.009	2.035	2.056	2.064
	∞	1.960	1.960	1.960	1.960	1.960	1.960	1.960
1 percent points	6	3.707	3.654	3.557	3.514	3.557	3.654	3.707
	8	3.707	3.643	3.495	3.363	3.307	3.328	3.355
$n_2 = 6$	12	3.707	3.636	3.453	3.246	3.104	3.053	3.055
	24	3.707	3.631	3.424	3.158	2.938	2.822	2.797
	∞	3.707	3.626	3.402	3.093	2.804	2.627	2.576
	6	3.355	3.328	3.307	3.363	3.495	3.643	3.707
	8	3.355	3.316	3.239	3.206	3.239	3.316	3.355
$n_2 = 8$	12	3.355	3.307	3.192	3.083	3.032	3.039	3.055
	24	3.355	3.301	3.158	2.988	2.862	2.805	2.797
	∞	3.355	3.295	3.132	2.916	2.723	2.608	2.576
	6	3.055	3.053	3.104	3.246	3.453	3.636	3.707
	8	3.055	3.039	3.032	3.083	3.192	3.307	3.355
$n_2 = 12$	12	3.055	3.029	2.978	2.954	2.978	3.029	3.055
	24	3.055	3.020	2.938	2.853	2.803	2.793	2.797
	∞	3.055	3.014	2.909	2.775	2.661	2.595	2.576
	6	2.797	2.822	2.938	3.158	3.424	3.631	3.707
	8	2.797	2.805	2.862	2.988	3.158	3.301	3.355
$n_2 = 24$	12	2.797	2.793	2.803	2.853	2.938	3.020	3.055
	24	2.797	2.785	2.759	2.747	2.759	2.785	2.797
	∞	2.797	2.777	2.726	2.664	2.613	2.585	2.576
	6	2.576	2.627	2.804	3.093	3.402	3.626	3.707
	8	2.576	2.608	2.723	2.916	3.132	3.295	3.355
$n_2 = \infty$	12	2.576	2.595	2.661	2.775	2.909	3.014	3.055
	24	2.576	2.585	2.613	2.664	2.726	2.777	2.797
	∞	2.576	2.576	2.576	2.576	2.576	2.576	2.576

This table has been taken from STATISTICAL TABLES by Sir Ronald A. Fisher and Frank Yates, 1953.

ANNEX E

(*Clause* 8.1)

CRITICAL VALUES OF THE *F*-DISTRIBUTION

Significance Level 0.005

(For one-sided test)

Degrees of Freedom $(n_1-1)\rightarrow$ (n_2-1) ↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	30	40	50	60	80	100	∞
1	161	200	216	225	230	234	237	239	241	242	243	244	245	245	246	246	247	247	248	248	250	251	252	252	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70	8.69	8.68	8.67	8.67	8.66	8.62	8.59	8.58	8.57	8.56	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86	5.84	5.83	5.82	5.81	5.80	5.75	5.72	5.70	5.69	5.67	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62	4.60	4.59	4.58	4.57	4.56	4.50	4.46	4.44	4.43	4.41	4.41	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94	3.92	3.91	3.90	3.88	3.87	3.81	3.77	3.75	3.74	3.72	3.71	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51	3.49	3.48	3.47	3.46	3.44	3.38	3.34	3.32	3.30	3.29	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22	3.20	3.19	3.17	3.16	3.15	3.08	3.04	3.02	3.01	2.99	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	2.05	3.03	3.01	2.99	2.97	2.96	2.95	2.94	2.86	2.83	2.80	2.79	2.77	2.76	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85	2.83	2.81	2.80	2.78	2.77	2.70	2.66	2.64	2.62	2.60	2.59	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72	2.70	2.69	2.67	2.66	2.65	2.57	2.53	2.51	2.49	2.47	2.46	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62	2.60	2.58	2.57	2.56	2.54	2.47	2.43	2.40	2.38	2.36	2.35	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53	2.51	2.50	2.48	2.47	2.46	2.38	2.34	2.31	2.30	2.27	2.26	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46	2.44	2.43	2.41	2.40	2.39	2.31	2.27	2.24	2.22	2.20	2.19	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40	2.38	2.37	2.35	2.34	2.33	2.25	2.20	2.18	2.16	2.14	2.12	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35	2.33	2.32	2.30	2.29	2.28	2.19	2.15	2.12	2.11	2.08	2.07	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31	2.29	2.27	2.26	2.24	2.23	2.15	2.10	2.08	2.06	2.03	2.02	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27	2.25	2.23	2.22	2.20	2.19	2.11	2.06	2.04	2.02	1.99	1.98	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23	2.21	2.20	2.18	2.17	2.16	2.07	2.03	2.00	1.98	1.96	1.94	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.22	2.20	2.18	2.17	2.15	2.14	2.12	2.04	1.99	1.97	1.95	1.92	1.91	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.11	2.10	2.01	1.96	1.94	1.92	1.89	1.88	181
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15	2.13	2.11	2.10	2.08	2.07	1.98	1.94	1.91	1.89	1.86	1.85	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.23	2.20	2.18	2.15	2.13	2.11	2.09	2.07	2.06	2.05	1.96	1.91	1.88	1.86	1.84	1.82	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18	2.15	2.13	2.11	2.09	2.07	2.05	2.04	2.03	1.94	1.89	1.86	1.84	1.82	1.80	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16	2.14	2.11	2.09	2.07	2.05	2.04	2.02	2.01	1.92	1.87	1.84	1.82	1.80	1.78	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.12	2.09	2.07	2.05	2.03	2.02	2.00	1.99	1.90	1.85	1.82	1.80	1.78	1.76	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.17	2.13	2.10	2.08	2.06	2.04	2.02	2.00	1.99	1.97	1.88	1.84	1.81	1.79	1.76	1.74	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	2.09	2.06	2.04	2.02	2.00	1.99	1.97	1.96	1.87	1.82	1.79	1.77	1.74	1.73	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.08	2.05	2.03	2.01	1.99	1.97	1.96	1.94	1.85	1.81	1.77	1.75	1.73	1.71	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01	1.99	1.98	1.96	1.95	1.93	1.84	1.79	1.76	1.74	1.71	1.70	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92	1.90	1.89	1.87	1.85	1.84	1.74	1.69	1.66	1.64	1.61	1.59	1.51
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95	1.92	1.89	1.87	1.85	1.83	1.81	1.80	1.78	1.69	1.63	1.60	1.58	1.54	1.52	1.44
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.93	1.89	1.86	1.84	1.82	1.80	1.78	1.76	1.75	1.65	1.59	1.56	1.53	1.50	148	1.39
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.93	1.89	1.86	1.84	1.81	1.79	1.77	1.75	1.74	1.72	1.62	1.57	1.53	1.50	1.47	1.45	1.35
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.91	1.88	1.84	1.82	1.79	1.77	1.75	1.73	1.72	1.70	1.60	1.54	1.51	1.48	1.45	1.43	1.32
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.90	1.86	1.83	1.80	1.78	1.76	1.74	1.72	1.70	1.69	1.59	1.53	1.49	1.46	1.43	1.41	1.30
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85	1.82	1.79	1.77	1.75	1.73	1.71	1.69	1.68	1.57	1.52	1.48	1.45	1.41	1.39	1.28
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75	1.72	1.69	1.67	1.64	1.62	1.60	1.59	1.57	1.46	1.39	1.35	1.32	1.27	1.24	1.00

NOTE — $(n_1 - 1)$ refers to the degrees of freedom for the larger mean square placed in the numerator.

ANNEX E — *Continued*

CRITICAL VALUES OF THE *F*-DISTRIBUTION

Significance Level 0.01

(For one-sided test)

Degrees of Freedom $(n_1-1)\rightarrow$ (n_2-1) ↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	30	40	50	60	80	100	∞
1	4050	5000	5400	5630	5760	5860	5930	5980	6020	6060	6080	6110	6130	6140	6160	6170	6180	6190	6200	6210	6260	6290	6300	6310	6330	6330	6370
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1	27.0	26.9	26.9	26.8	26.8	26.8	26.7	26.7	26.5	26.4	26.4	26.3	26.3	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4	14.3	14.2	14.2	14.2	14.1	14.1	14.0	14.0	13.8	13.7	13.7	13.7	13.6	13.6	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89	9.85	9.77	9.72	9.68	9.64	9.61	9.58	9.55	9.38	9.29	9.24	9.20	9.16	9.13	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72	7.66	7.60	7.56	7.52	7.48	7.45	7.42	7.40	7.23	7.14	7.09	7.06	7.01	6.99	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47	6.41	6.36	6.31	6.27	6.24	6.21	6.18	6.16	5.99	5.91	5.86	5.82	5.78	5.75	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67	5.61	5.56	5.52	5.48	5.44	5.41	5.38	5.36	5.20	5.12	5.07	5.03	4.99	4.96	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11	5.05	5.00	4.96	4.92	4.89	4.86	4.83	4.81	4.65	4.57	4.52	4.48	4.44	4.42	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71	4.65	4.60	4.56	4.52	4.49	4.46	4.43	4.41	4.25	4.17	4.12	4.08	4.04	4.01	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40	4.34	4.29	4.25	4.21	4.18	4.15	4.12	4.10	3.94	3.86	3.81	3.78	3.73	3.71	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16	4.10	4.05	4.01	3.97	3.94	3.91	3.88	3.86	3.70	3.62	3.57	3.54	3.49	3.47	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96	3.91	3.86	3.82	3.78	3.75	3.72	3.69	3.66	3.51	3.43	3.38	3.34	3.30	3.27	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.86	3.80	3.75	3.70	3.66	3.62	3.59	3.56	3.53	3.51	3.35	3.27	3.22	3.18	3.14	3.11	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67	3.61	3.56	3.52	3.49	3.45	3.42	3.40	3.37	3.21	3.13	3.08	3.05	3.00	2.98	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55	3.50	3.45	3.41	3.37	3.34	3.31	3.28	3.26	3.10	3.02	2.97	2.93	2.89	2.86	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46	3.40	3.35	3.31	3.27	3.24	3.21	3.18	3.16	3.00	2.92	2.87	2.83	2.79	2.76	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.79	3.60	3.51	3.43	3.37	3.32	3.27	3.23	3.19	3.16	3.13	3.10	3.08	2.92	2.84	2.78	2.75	2.70	2.68	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30	3.24	3.19	3.15	3.12	3.08	3.05	3.03	3.00	2.84	2.76	2.71	2.67	2.63	2.60	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23	3.18	3.13	3.09	3.05	3.02	2.99	2.96	2.94	2.78	2.69	2.64	2.61	2.56	2.54	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.24	3.17	3.12	3.07	3.03	2.99	2.96	2.93	2.90	2.88	2.72	2.64	2.58	2.55	2.50	2.48	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12	3.07	3.02	2.98	2.94	2.91	2.88	2.85	2.83	2.67	2.58	2.53	2.50	2.45	2.42	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.14	3.07	3.02	2.97	2.93	2.89	2.86	2.83	2.80	2.78	2.62	2.54	2.48	2.45	2.40	2.37	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03	2.98	2.93	2.89	2.85	2.82	2.79	2.76	2.74	2.58	2.49	2.44	2.40	2.36	2.33	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	3.06	2.99	2.94	2.89	2.85	2.81	2.78	2.75	2.72	2.70	2.54	2.45	2.40	2.36	2.32	2.29	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96	2.90	2.86	2.82	2.78	2.74	2.72	2.69	2.66	2.50	2.42	2.36	2.33	2.28	2.25	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.99	2.93	2.87	2.82	2.78	2.75	2.71	2.68	2.66	2.63	2.47	2.38	2.33	2.29	2.25	2.22	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90	2.84	2.79	2.75	2.72	2.68	2.65	2.63	2.60	2.44	2.35	2.30	2.26	2.22	2.19	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.93	2.87	2.81	2.77	2.73	2.69	2.66	2.63	2.60	2.57	2.41	2.33	2.27	2.23	2.19	2.16	2.03
30	7.56	5.39	4.51	4.02	3.72	3.47	3.30	3.17	3.07	3.98	2.91	2.84	2.79	2.74	2.70	2.66	2.63	2.60	2.57	2.55	2.39	2.30	2.25	2.21	2.16	2.13	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66	2.61	2.56	2.52	2.48	2.45	2.42	2.39	2.37	2.20	2.11	2.06	2.02	1.97	1.94	1.80
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.79	2.70	2.63	2.56	2.51	2.46	2.42	2.38	2.35	2.32	2.29	2.27	2.10	2.01	1.95	1.91	1.86	1.82	1.68
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50	2.44	2.39	2.35	2.31	2.28	2.25	2.22	2.20	2.03	1.94	1.88	1.84	1.78	1.75	1.60
70	7.01	4.92	4.08	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.51	2.45	2.40	2.35	2.31	2.27	2.23	2.20	2.18	2.15	1.98	1.89	1.83	1.78	1.73	1.70	1.54
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.48	2.42	2.36	2.31	2.27	2.23	2.20	2.17	2.14	2.12	1.94	1.85	1.79	1.75	1.69	1.66	1.49
90	6.93	4.85	4.01	3.54	3.23	3.01	2.84	2.72	2.61	2.52	2.45	2.39	2.33	2.29	2.24	2.21	2.17	2.14	2.11	2.09	1.92	1.82	176	1.72	1.66	1.62	1.46
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.43	2.37	2.31	2.26	2.22	2.19	2.15	2.12	2.09	2.07	1.89	1.80	1.73	1.69	1.63	1.60	1.43
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18	2.13	2.08	2.04	2.00	1.97	1.93	1.90	1.88	1.70	1.59	1.52	1.47	1.40	1.36	1.00

NOTE — $(n_1 - 1)$ refers to the degrees of freedom for the larger mean square placed in the numerator.

ANNEX E — *Continued*

CRITICAL VALUES OF THE *F*-DISTRIBUTION

Significance Level 0.05

(For two-sided test)

Degrees of Freedom (n_1-1)→ (n_2-1) ↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	30	40	50	60	80	100	∞
1	648	800	864	900	922	937	948	957	963	969	973	977	980	983	985	987	989	990	992	993	1001	1006	1008	1010	1012	1013	1018
2	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5	39.5	39.5
3	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.4	14.4	14.3	14.3	14.3	14.3	14.2	14.2	14.2	14.2	14.2	14.1	14.0	14.0	14.0	14.0	14.0	13.9
4	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.79	8.75	8.72	8.69	8.66	8.64	8.62	8.60	8.58	8.56	8.46	8.41	8.38	8.36	8.33	8.32	8.26
5	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.57	6.52	6.49	6.46	6.43	6.41	6.39	6.37	6.35	6.33	6.23	6.18	6.14	6.12	6.10	6.08	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.41	5.37	5.33	5.30	5.27	5.25	5.23	5.21	5.19	5.17	5.07	5.01	4.98	4.96	4.93	4.92	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.71	4.67	4.63	4.60	4.57	4.54	4.52	4.50	4.48	4.47	4.36	4.31	4.28	4.25	4.23	4.21	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.24	4.20	4.16	4.13	4.10	4.08	4.05	4.03	4.02	4.00	3.89	3.84	3.81	3.78	3.76	3.74	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.91	3.87	3.83	3.80	3.77	3.74	3.72	3.70	3.68	3.67	3.56	3.51	3.47	3.45	3.42	3.40	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.66	3.62	3.58	3.55	3.52	3.50	3.47	3.45	3.44	3.42	3.31	3.26	3.22	3.20	3.17	3.15	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.47	3.43	3.39	3.36	3.33	3.30	3.28	3.26	3.24	3.23	3.12	3.06	3.03	3.00	2.97	2.96	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.32	3.28	3.24	3.21	3.18	3.15	3.13	3.11	3.09	3.07	2.96	2.91	2.87	2.85	2.82	2.80	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.20	3.15	3.12	3.08	3.05	3.03	3.00	2.98	2.96	2.95	2.84	2.78	2.74	2.72	2.69	2.67	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.09	3.05	3.01	2.98	2.95	2.92	2.90	2.88	2.86	2.84	2.73	2.67	2.64	2.61	2.58	2.56	2.49
15	6.20	4.76	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	3.01	2.96	2.92	2.89	2.86	2.84	2.81	2.79	2.77	2.76	2.64	2.58	2.55	2.52	2.49	2.47	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.93	2.89	2.85	2.82	2.79	2.76	2.74	2.72	2.70	2.63	2.57	2.51	2.47	2.45	2.42	2.40	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.87	2.82	2.79	2.75	2.72	2.70	2.67	2.65	2.63	2.62	2.50	2.44	2.41	2.38	2.35	2.33	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.81	2.77	2.73	2.70	2.67	2.64	2.62	2.60	2.58	2.56	2.44	2.38	2.35	2.32	2.29	2.27	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.76	2.72	2.68	2.65	2.62	2.59	2.57	2.55	2.53	2.51	2.39	2.33	2.30	2.27	2.24	2.22	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.72	2.68	2.64	2.60	2.57	2.55	2.52	2.50	2.48	2.46	2.35	2.29	2.25	2.22	2.19	2.17	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.68	2.64	2.60	2.56	2.53	2.51	2.48	2.46	2.44	2.42	2.31	2.25	2.21	2.18	2.15	2.13	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.65	2.60	2.56	2.53	2.50	2.47	2.45	2.43	2.41	2.39	2.27	2.21	2.17	2.14	2.11	2.09	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.62	2.57	2.53	2.50	2.47	2.44	2.42	2.39	2.37	2.36	2.24	2.18	2.14	2.11	2.08	2.06	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.59	2.54	2.50	2.47	2.44	2.41	2.39	2.36	2.35	2.33	2.21	2.15	2.11	2.08	2.05	2.02	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.56	2.51	2.48	2.44	2.41	2.38	2.36	2.34	2.32	2.30	2.18	2.12	2.08	2.05	2.02	2.00	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.54	2.49	2.45	2.42	2.39	2.36	2.34	2.31	2.29	2.28	2.16	2.09	2.05	2.03	1.99	1.97	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.51	2.47	2.43	2.39	2.36	2.34	2.31	2.29	2.27	2.25	2.13	2.07	2.03	2.00	1.97	1.94	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.49	2.45	2.41	2.37	2.34	2.32	2.29	2.27	2.25	2.23	2.11	2.05	2.01	1.98	1.94	1.92	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.48	2.43	2.39	2.36	2.32	2.30	2.27	2.25	2.23	2.21	2.09	2.03	1.99	1.96	1.92	1.90	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	5.65	2.57	2.51	2.46	2.41	2.37	2.34	2.31	2.28	2.26	2.23	2.21	2.20	2.07	2.01	1.97	1.94	1.90	1.88	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.33	2.29	2.25	2.21	2.18	2.15	2.13	2.11	2.09	2.07	1.94	1.88	1.83	1.80	1.76	1.74	1.64
50	5.34	3.98	3.39	3.06	2.83	2.67	2.55	2.46	2.38	2.32	2.26	2.22	2.18	2.14	2.11	2.08	2.06	2.03	2.01	1.99	1.87	1.80	1.75	1.72	1.68	1.66	1.55
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.22	2.17	2.13	2.09	2.06	2.03	2.01	1.98	1.96	1.94	1.82	1.74	1.70	1.67	1.62	1.60	1.48
70	5.25	3.89	3.31	2.98	2.75	2..60	2.48	2.38	2.30	2.24	2.18	2.14	2.10	2.06	2.03	2.00	1.97	1.95	1.93	1.91	1.78	1.71	1.66	1.63	1.58	1.56	1.44
80	5.22	3.86	3.28	2.95	2.73	2.57	2.45	2.36	2.28	2.21	2.16	2.11	2.07	2.03	2.00	1.97	1.95	1.93	1.90	1.88	1.75	1.68	1.63	1.60	1.55	1.53	1.40
90	5.20	3.84	3.27	2.93	2.71	2.55	2.43	2.34	2.26	2.19	2.14	2.09	2.05	2.02	1.98	1.95	1.93	1.91	1.88	1.86	1.73	1.66	1.61	1.58	1.53	1.50	1.37
100	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24	2.18	2.12	2.08	2.04	2.00	1.97	1.94	1.91	1.89	1.87	1.85	1.71	1.64	1.59	1.56	1.51	1.48	1.35
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.99	1.94	1.90	1.87	1.83	1.80	1.78	1.75	1.73	1.71	1.57	1.48	1.43	1.39	1.33	1.30	1.00

NOTE — $(n_1 - 1)$ refers to the degrees of freedom for the larger mean square placed in the numerator.

ANNEX E — *Concluded*

CRITICAL VALUES OF THE *F*-DISTRIBUTION

Significance Level 0.01

(For two-sided test)

Degrees of Freedom $(n_1-1) \rightarrow$ (n_2-1) ↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	30	40	50	60	80	100	∞
1	16 200	20 000	21 600	22 500	23 100	23 400	23 700	23 900	24 100	24 200	24 300	24 400	24 500	24 600	24 600	24 700	24 700	24 800	24 800	24 800	25 000	25 100	25 200	25 300	25 300	25 300	25 500
2	198	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	200
3	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.5	43.4	43.3	43.2	43.1	43.0	42.9	42.9	42.8	42.8	42.5	42.3	42.2	42.1	42.1	42.0	41.8
4	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.8	20.7	20.6	20.5	20.4	20.4	20.3	20.3	20.2	20.2	19.9	19.8	19.7	19.6	19.5	19.5	19.3
5	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.5	13.4	13.3	13.2	13.1	13.1	13.0	13.0	12.9	12.9	12.7	12.5	12.5	12.4	12.3	12.3	12.1
6	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.1	10.0	9.95	9.88	9.81	9.76	9.71	9.66	9.62	9.59	9.36	9.21	9.17	9.12	9.06	9.03	8.88
7	16.2	12.4	10.0	10.0	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18	8.10	8.03	7.97	7.93	7.87	7.83	7.79	7.75	7.53	7.42	7.35	7.31	7.25	7.22	7.08
8	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01	6.94	6.87	6.81	6.76	6.72	6.68	6.64	6.61	6.40	6.29	6.22	6.18	6.12	6.09	5.95
9	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23	6.15	6.09	6.03	5.98	5.94	5.90	5.86	5.83	5.62	5.52	5.45	5.41	5.36	5.32	5.19
10	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66	5.59	5.53	5.47	5.42	5.38	5.34	5.30	5.27	5.07	4.97	4.90	4.86	4.80	4.77	4.64
11	12.2	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24	5.16	5.10	5.05	5.00	4.96	4.92	4.89	4.86	4.65	4.55	4.49	4.44	4.39	4.36	4.23
12	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91	4.84	4.77	4.72	4.67	4.63	4.59	4.56	4.53	4.33	4.23	4.17	4.12	4.07	4.04	3.90
13	11.4	8.19	6.93	6.23	5.79	5.48	5.25	5.08	4.94	4.82	4.72	4.64	4.57	4.51	4.46	4.41	4.37	4.33	4.30	4.27	4.07	3.97	3.91	3.87	3.81	3.78	3.65
14	11.1	7.92	6.68	6.00	5.56	5.26	5.03	4.86	4.72	4.60	4.51	4.43	4.36	4.30	4.25	4.20	4.16	4.12	4.09	4.06	3.86	3.76	3.70	3.66	3.60	3.57	3.44
15	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25	4.18	4.12	4.07	4.02	3.98	3.95	3.91	3.88	3.69	3.58	3.52	3.48	3.43	3.39	3.26
16	10.6	7.51	6.30	5.64	5.21	4.91	4.69	4.52	4.38	4.27	4.18	4.10	4.03	3.97	3.92	3.87	3.83	3.80	3.76	3.73	3.54	3.44	3.37	3.33	3.28	3.25	3.11
17	10.4	7.35	6.16	5.50	5.07	4.78	4.56	4.39	4.25	4.14	4.05	3.97	3.90	3.84	3.79	3.75	3.71	3.67	3.64	3.61	3.41	3.31	3.25	3.21	3.15	3.12	2.98
18	10.2	7.21	6.03	5.37	4.96	4.66	4.44	4.28	4.14	4.03	3.94	3.86	3.79	3.73	3.68	3.64	3.60	3.56	3.53	3.50	3.30	3.20	3.14	3.10	3.04	3.01	2.87
19	10.1	7.09	5.92	5.27	4.85	4.56	4.34	4.18	4.04	3.93	3.84	3.76	3.70	3.64	3.59	3.54	3.50	3.46	3.43	3.40	3.21	3.11	3.04	3.00	2.95	2.91	2.78
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68	3.61	3.55	3.50	3.46	3.42	3.38	3.35	3.32	3.12	3.02	2.96	2.92	2.86	2.83	2.69
21	9.83	6.89	5.73	5.09	4.68	4.39	4.18	4.01	3.88	3.77	3.68	3.60	3.54	3.48	3.43	3.38	3.34	3.31	3.27	3.24	3.05	2.95	2.88	2.84	2.78	2.75	2.61
22	9.73	6.81	5.65	5.02	4.61	4.32	4.11	3.94	3.81	3.70	3.61	3.54	3.47	3.41	3.36	3.31	3.27	3.24	3.20	3.18	2.98	2.88	2.82	2.77	2.72	2.69	2.55
23	9.63	6.73	5.58	4.95	4.54	4.26	4.05	3.88	3.75	3.64	3.55	3.47	3.41	3.35	3.30	3.25	3.21	3.18	3.15	3.12	2.92	2.82	2.76	2.71	2.66	2.62	2.48
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42	3.35	3.30	3.25	3.20	3.16	3.12	3.09	3.06	2.87	2.77	2.70	2.66	2.60	2.57	2.43
25	9.48	6.60	5.46	4.84	4.43	4.15	3.94	3.78	3.64	3.54	3.45	3.37	3.30	3.25	3.20	3.15	3.11	3.08	3.04	3.01	2.82	2.72	2.65	2.61	2.55	2.52	2.38
26	9.41	6.54	5.41	4.79	4.38	4.10	3.89	3.73	3.60	3.49	3.40	3.33	3.26	3.20	3.15	3.11	3.07	3.03	3.00	2.97	2.77	2.67	2.61	2.56	2.51	2.47	2.33
27	9.34	6.49	5.36	4.74	4.34	4.06	3.85	3.69	3.56	3.45	3.36	3.28	3.22	3.16	3.11	3.07	3.03	2.99	2.96	2.93	2.73	2.63	2.57	2.52	2.47	2.43	2.29
28	9.28	6.44	5.32	4.70	4.30	4.02	3.81	3.65	3.52	3.41	3.32	3.25	3.18	3.12	3.07	3.03	2.99	2.95	2.92	2.89	2.69	2.59	2.53	2.48	2.43	2.39	2.25
29	9.23	6.40	5.28	4.66	4.26	3.98	3.77	3.61	3.48	3.38	3.29	3.21	3.15	3.09	3.04	2.99	2.95	2.92	2.88	2.86	2.66	2.56	2.49	2.45	2.39	2.36	2.21
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18	3.11	3.06	3.01	2.96	2.92	2.89	2.85	2.82	2.63	2.52	2.46	2.42	2.36	2.32	2.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95	2.89	2.83	2.78	2.74	2.70	2.66	2.63	2.60	2.40	2.30	2.23	2.18	2.12	2.09	1.93
50	8.63	5.90	4.83	4.23	3.85	3.58	3.38	3.22	3.09	2.99	2.90	2.82	2.76	2.70	2.65	2.61	2.57	2.53	2.50	2.47	2.27	2.16	2.10	2.05	1.99	1.95	1.79
60	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74	2.68	2.62	2.57	2.53	2.49	2.45	2.42	2.39	2.19	2.08	2.01	1.96	1.90	1.86	1.69
70	8.40	5.72	4.65	4.08	3.70	3.43	3.23	3.08	2.95	2.85	2.76	2.68	2.62	2.56	2.51	2.47	2.43	2.39	2.36	2.33	2.13	2.02	1.95	1.90	1.84	1.80	1.62
80	8.33	5.67	4.61	4.03	3.65	3.39	3.19	3.03	2.91	2.80	2.72	2.64	2.58	2.52	2.47	2.43	2.39	2.35	2.32	2.29	2.08	1.97	1.90	1.85	1.79	1.75	1.56
90	8.28	5.62	4.57	3.99	3.62	3.35	3.15	3.00	2.87	2.77	2.68	2.61	2.54	2.49	2.44	2.39	2.35	2.32	2.28	2.25	2.05	1.94	1.87	1.82	1.75	1.71	1.52
100	8.24	5.59	4.54	3.96	3.59	3.33	3.13	2.97	2.85	2.74	2.66	2.58	2.52	2.46	2.41	2.37	2.33	2.29	2.26	2.23	2.02	1.91	1.84	1.79	1.72	1.68	1.49
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36	2.29	2.24	2.19	2.14	2.10	2.06	2.03	2.00	1.79	1.67	1.59	1.53	1.45	1.40	1.00

NOTE — $(n_1 - 1)$ refers to the degrees of freedom for the larger mean square placed in the numerator.

ANNEX F

(*Foreword*)

COMMITTEE COMPOSITION

Statistical Methods for Quality and Reliability Sectional Committee, MSD 3

Organization	*Representative(s)*
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